Bulgarian Academy of Sciences. Space Research Institute.
Aerospace Research in Bulgaria. 18. 2003. Sofia

SOME FEATURES OF α DISC AND ADVECTIVE-DOMINATED ACCRETION DISC. SELF-SIMILAR SOLUTIONS AND THEIR COMPARISON -II

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Abstract

Using the models from part I, we have derived the basic parameters, describing the discs. We have obtained the self-similar solutions of the evolution for both types - ADAD and α discs. The results are expressed quantitatively to demonstrate our conclusion.

1. Introduction

As a continuation of Part I, dedicated to the priority of advection theory and the properties of advection - dominated flow and the comparison with standard accretion theory, here we present the actual results of our calculations. In many problems, the simple self-similar solutions don't correspond to complete solution [12]. They are intervening asymptotically and in a number of cases they give a sufficient idea of the studied physical phenomena with correct boundary conditions. As a result of the required transformation performed in our letter and using work [7] with adequate variables, we will obtain self-similar solutions, too.

2. Equations, describing the evolution of α disc and ADAD.

The last two systems from part I [8] (eq. 3.29 ÷ 3.33; 3.34 ÷ 3.38) enable us to obtain all parameters of the disc, so, we are looking only for Σ in explicit form. To this end we will use the conservation laws; using (eq. 2.10, see [8]) we obtain:

$$(II.1) \quad \Sigma V_r r = \frac{\overset{o}{\dot{M}}}{2\pi} = -\left(\frac{\partial h_*}{\partial h}\right)^{-1} \frac{\partial F}{\partial h}$$

which gives respectively:

$$(II.2)\quad \dot{M} = -2\pi \frac{\partial F}{\partial h}$$

$$(II.3)\quad \dot{M} = -\frac{2\pi}{c_2} \frac{\partial F}{\partial h}$$

From (eq. 2.9, see [8]) and (II.1) come after:

$$(II.4)\quad \frac{\partial \Sigma}{\partial t} = \frac{1}{2} \frac{(GM)^2}{L^3} \frac{\partial}{\partial h} \left\{ \left(\frac{\partial h_*}{\partial h} \right)^{-1} \frac{\partial F}{\partial h} \right\}$$

as we apply (eq. (3.33) and eq. (3.38) see [8]) and the relation $\frac{\overline{\nu}}{h} = \left(\frac{2\alpha c_3}{3c_2} \right)$ we obtain the follows diffusion equations:

$$(II.5)\quad \frac{\partial F}{\partial t} = \Pi \frac{F^m}{h^n} \frac{\partial^2 F}{\partial h^2}$$

$$(II.6)\quad \frac{\partial F}{\partial t} = \frac{\Pi_a}{h} \frac{\partial^2 F}{\partial h^2}$$

where:

$$\Pi = \frac{AF^{\frac{1}{A}}}{2} (GM)^2 \qquad \Pi_a = \frac{\alpha}{2} \frac{c_3}{c_2} (GM)^2$$

$$m = \frac{4 + 2a_1}{10 + 2a_1 - 2b_1 - c_1} \qquad n = \frac{12 + 6a_1 + 2b_1 - 5c_1}{10 + 2a_1 - 2b_1 - c_1}$$

From (II.4) we get:

$$(II.7)\quad \Sigma = \frac{(GM)^2 F^{1-m}}{2(1-m)\Pi h^{3-n}}$$

$$(II.8)\quad \Sigma = \frac{(GM)^2}{2\Pi_a} \frac{F}{h^2}$$

3. Self – Similar Solutions.

First we will define the role of self-similar solutions and then we will give an example for their application. Such an example is the examination of the temperature diffusion equation for stationary conductive medium, presented in [7]:

$$\frac{\partial T}{\partial t} = D\nabla T$$

D - diffusion constant.

We will determine the temperature at successive moments of time, when the initial distribution is: $T = Kr^x$, r - the distance to the centre of the coordinate system.

If we define the scale of the temperature U, the distance Λ and the time η, then we can determine the dimensions D and K:

$[D] = \eta^{-1}\Lambda^2$ and $[K] = \Lambda^{-x}U$

D independent of U

Sometimes, after the beginning of the process, the typical length scale depending on time may be defined as:

$$\Lambda_c(t) = (Dt)^{1/2}$$

The time-dependant temperature scale may be defined in a similar way:

$$T_c(t) = K\Lambda_c(t)^x$$

The solution should yield T as a function of t and r. In non-dimensional form:

$$\frac{T}{T_c} = \frac{T}{K\Lambda^x}$$

This form should be a function of $\frac{r}{\Lambda_l(t)}$ and $\frac{t}{t}$.

So we obtain the solution in the form :

$$T = K\Lambda_c{}^x T_*\left(\frac{r}{\Lambda_c(t)}\right)$$

T_* - dimensionless function composed of its dimensionless arguments.

The obtained result is a self-similar solution, since time dependent scales are used. The temperature scale ia always the function of scale featuring the length. This is the self-similarity of the problem which denotes that variable scales of Λ_c and T_c may be selected. Because of this, it is possible to represent the scale of characteristics by a single variable function.

Therefore, the presence of several dimensions for the independent constants, including the boundary conditions of the problem, defines the neccessity of self-similar solution.

Let us examine the problem where the self-similar solution is of the first order. The time behavior of a thin disc is defined by (II.5) under the assumption that for the initial moment $t = 0$ the distribution is:

$$F = Kh^y$$

The dimensions of all values in (II.5) and initial conditions are:

$$[h] = \Lambda^2\eta^{-1}; [t] = \eta; [F] = M\Lambda^2\eta^{-2};$$

$$[\Pi] = M^{-m}\Lambda^{-2(n-m+2)}\eta^{2n-m-3};$$

$$[K] = M\Lambda^{2(1-y)}\eta^{y-2}.$$

Now we have to determine the typical scale of the total angular momentum $h_c(t)$ and typical scale of friction $F_c(t)$ for each moment $t > 0$.

The first value is obtained from the dimensional analysis of (II.5):

$$h_c(t) = \left(\Pi F_c(t)^m t\right)\frac{1}{n+2}$$

For $F_c(t)$ we use the initial distribution:

$$F_c(t) = Kh_c(t)^y$$

Subtituting the last equation in the upper one, we obtain for h_c:

$$h_c(t) = \left(\Pi K^m t\right)\frac{1}{n+2-xm}.$$

The solution of the problem yields F as a function of h and t and may be expressed in dimensionless form:

$$\frac{F}{F_c} = \frac{F}{Kh_c(t)^y} = F_*\left(\frac{h}{h_c}, \frac{t}{t}\right) = F_*\left(\frac{h}{h_c}\right)$$

Then function F will take the form:

$$F(h.t) = Kh_c^{\,y}(t)F_*\left(\frac{h}{h_c}\right).$$

Using the present above [7], we divide the variables in (II.5) and (II.6):

$$F(th) = F(t)f(\xi), \ \xi = \frac{h}{h_0} = \sqrt{\frac{r}{r_{out}}}; \quad h_0 = \sqrt{GMr_{out}}$$

r_{out} is the edge of the disc.
Then:

$$(\text{III.1}) \quad F(t) = \left[\frac{h_0^{\,n+2}}{-\lambda m\Pi(t+t_0)} \right]^{\frac{1}{m}}$$

$$(\text{III.2}) \quad F(t) = F_0 e^{\beta t}; \quad \beta = \frac{\lambda_a \Pi_a}{h_0^{\,3}}$$

$$(\text{III.3}) \quad \frac{\partial^2 f}{\partial \xi^2} = \lambda \xi^n f^{1-m}$$

$$(\text{III.4}) \quad \frac{\partial^2 f}{\partial \xi^2} = \lambda_a \xi f$$

Also, we can search the function in polynomial form:

$$(\text{III.5}) \quad f(\xi) = a_0 \xi + a' \xi^l + a_2 \xi^{l_1}$$

$$l = 3 + n - m, \quad a' = \frac{\lambda a_0^{\,1-m}}{l(l-1)}, \quad a_2 = \frac{\lambda^2 a_0^{\,1-2m}(1-m)}{2l(2l-1)(l-1)^2}$$

$$a_0 = \frac{2l+1}{2(l-1)}, \quad \lambda = l(2l-1)a_0^{\,m-1} - 2l(l-1)a_0^{\,m}$$

and the boundary conditions are:

$$f(1) = 1 \qquad f(0) = 0$$
$$f'(1) = 0$$

finally we replace (III.1), (III.2), (III.5) in (II.7) and (II.8)

Then we replace the result in ((eq. 3.29 ÷ 3.32,) and (eq. 3.34 ÷ 3.37), see [8]). As a result, the parameters of two discs are obtained in explicit form of time and dimensionless coordinate ξ.

Standard accretion disc:

$$\dot{M} = \frac{\dot{M}_k f'}{\Psi^{\frac{1}{m}}}; \qquad \dot{M}_k = \frac{-2\pi}{h_0}\left(\frac{h_0^{\,n+2}}{-\lambda m \Pi t_\phi} \right)^{\frac{1}{m}}; \quad \Psi = \frac{t+t_0}{t_\phi}$$

$$\frac{\Sigma}{\Sigma_k} = \Psi^{\frac{m-1}{m}} f^{1-m} \xi^{n-3}; \qquad \Sigma_k = \frac{1}{2}\frac{(GM)^2}{h_0^{\,3-n}} \frac{1}{\Pi(1-m)} \left(\frac{h^{n-2}}{-\lambda m \Pi t_\phi} \right)^{\frac{1-m}{m}}$$

$$\frac{T}{T_k} = \Psi^{2N_1 \frac{m-1}{m}} f^{2N_1(1-m)} \xi^{2N_1(n-3)-6N_2}; \qquad T_k = T_0 \Sigma_k^{\,2N_1} \omega_{k0}^{\,2N_2}$$

$$\omega_{k0} = \omega_k(r_{out})$$

(III.6)

$$\frac{V_s}{V_{sk}} = \Psi^{N_1\frac{m-1}{m}} f^{N_1(1-m)} \xi^{N_1(n-3)-3N_2}; \qquad V_{sk} = V_{s0}\Sigma_k^{N_1}\omega_{k0}^{N_2}$$

$$\frac{W_{r\varphi}}{W_{r\varphi}^{\ k}} = \Psi^{(2N_1+1)\frac{m-1}{m}} f^{(2N_1+1)(1-m)} \xi^{(2N_1+1)(n-3)-6N_2};$$

$$W_{r\varphi}^{\ k} = W_{r\varphi 0}\Sigma_k^{2N_1+1}\omega_{k0}^{2N_2}$$

$$\frac{P}{P_k} = \Psi^{(N_1+1)\frac{m-1}{m}} f^{(N_1+1)(1-m)} \xi^{(N_1+1)(n-3)-3(N_2+1)}; \qquad P_k = P_0\Sigma_k^{N_1+1}\omega_{k0}^{(N_2+1)}$$

$$\tau_k = k_1 T_0^{b_1} V_{s0}^{c_1} \Sigma_k^{q_1} \omega_k^{q_2}$$

$$\frac{\tau}{\tau_k} = \Psi^{q_1\frac{m-1}{m}} f^{(1-m)q_1} \xi^{q_1(n-3)-3q_2}; \quad q_1 = a_1 + 1 + (2b_1 + c_1)N_1$$

$$q_2 = (2b_1 + c_1)N_2 - c_1$$

$$L = \sigma\dot{M}(0,t)c^2; \quad \frac{L}{L_E} = \frac{\sigma\dot{M}_k[M_\circ / day]}{L_E}\frac{a_0 c^2}{\Psi^{\frac{1}{m}}}$$

<u>Advection - dominated discs:</u>

$$\dot{M} = \dot{M}_a e^{\beta t} f'; \qquad \dot{M}_a = \frac{-2\pi}{c_2}\frac{F_0}{h_0}$$

$$\frac{\Sigma}{\Sigma_a} = e^{\beta t} f \xi^{-2}; \qquad \Sigma_a = \frac{F_0}{c_3 \alpha h_0^{\ 2}}$$

$$\frac{T}{T_a} = \xi^{-2}; \qquad T_a = T_0^{\ a}\omega_{k0}^{\ 2} r_{out}^{\ 2}$$

$$\frac{V_s}{V_{sa}} = \xi^{-1}; \qquad V_{sa} = V_{s0}^{\ a}\omega_{k0} r_{out}$$

(III.7)

$$\frac{W_{r\varphi}}{W_{r\varphi a}} = e^{\beta t} f \xi^{-4}; \qquad W_{r\varphi a} = W_{r\varphi 0}^{\ a}\Sigma_a \omega_{k0}^{\ 2} r_{out}^{\ 2}$$

$$\frac{P}{P_a} = e^{\beta t} f \xi^{-6}; \qquad P_a = P_0^{\ a}\Sigma_0 \omega_{k0}^{\ 2} r_{out}$$

$$\frac{\tau}{\tau_a} = e^{\beta t} f \xi^{-2}; \qquad \tau_a = \chi_0 \Sigma_a$$

$$\frac{L}{L_E} = \sigma_a \frac{\dot{M}_a\left[M_\odot / day\right]}{L_E} a_a c^2 e^{\beta t}$$

Using Tables 1 and 2 (Appendix 1), and from (III.6) we obtain the parameters of the disc for two regimes: Thompson's opacity and free-to-free transition.

To obtain the parameters of the advective disc we must define the constants c_2, c_3. Using the equations

$$\omega = c_2 \omega_k ; \quad h = \omega_k r^2 ; \quad h_* = \omega r^2 ; \quad \frac{\partial h_*}{\partial h} = c_2$$

we can find c_2.

But the value of c_3 cannot be defined precisely, we can give an appreciation only and taking into account physically and mathematically conditions. To keep the slim disc formation it is necessary $\frac{H}{r}$ doesn't exceed 10^{-2}. But the disc is advective hot and then H/r is in maximum, that is why we consider $\frac{H}{r} = 10^{-2}$.

4. Comments

A comparison has been made between the standard and the advective model of the accretion disc and as a result, the main parameters of both discs in dimensionless quantities are obtained. We have used hydrodynamical equations, as we have added the terms describing the advection. The obtained solutions are self-similar.

The results (Appendix 2) lay down the field of action for the second theory. They prove that the new advective theory can be used, while the main advantage of the standard theory - the slim disc approximation, remains.

The presentation enables us to obtain the full approximation solution for disc parameters at non-stationary accretion. Although the self-similar solution doesn't fit in accurately, it displays good quality estimation for the physical processes in a given astrophysical disc.

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НЯКОИ ОСОБЕНОСТИ НА α ДИСК И АДВЕКТИВНО - ДОМИНИРАЩ АКРЕЦИОНЕН ДИСК. АВТОМОДЕЛНИ РЕШЕНИЯ И ТЯХНОТО СРАВНЕНИЕ - II

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Резюме

На базата на структурираните модели в част I, са изведени основните параметри, характеризиращи двата диска. Получени са автомоделни решения за еволюцията на двата типа - Адвективно-доминиращ и α диск. Резултатите са представени и количествено за да потвърдят нашите изводи.

APPENDIX 1

Table No.1

Regime	a_1	b_1	c_1
χ_0	0	0	0
χ_{ff}	1	-3,5	-1
α	M/M_o	r_{out}/R_0	μ
0,3	3	1	0,5

Table No.2

Regim ежим	m	n	λ	a_0	a'	a_2	l	l_1	N_1	N_2	q_1	q_2
χ_0	2/5	1,2	3,482	1,376	-0,39	0,02	3,8	6,6	1/3	1/6	1	0
χ_{ff}	0,3	0,8	3,137	1,430	-0,46	0.03	3,5	6,0	3/14	1/7	2/7	-1/7

Table No.3

c_1	c_3	λ_a	a_a	a"	a_3	γ	γ_1	$\beta\left[\frac{1}{d}\right]$
1	10^{-4}	-5,33	1,5	-0,66	0.08	4	7	$7{,}46.10^{-3}$

APPENDIX 2

Table No.4

Σ_T/Σ_k	Σ_{ff}/Σ_k	Σ/Σ_{ad}	ξ	$\lg\Sigma/\Sigma_k$	$\lg\Sigma/\Sigma_{ad}$	Δ
517,15		150,00	0,01	2,71	2,18	0,8
32,73		14,99	0,1	1,51	1,18	0,08
14,18		7,47	0,2	1,15	0,87	0,07
$\Delta=10^{-2}$	7,73	4,94	0,3	0,89	0,69	0,05
	4,96	3,64	0,4	0,70	0,56	0,04
	3,49	2,93	0,5	0,54	0,47	0,03
	2,59	2,27	0,6	0,41	0,36	0,03
	1,99	1,83	0,7	0,30	0,26	0,03
	1,57	1,48	0,8	0,19	0,17	0,03
	1,25	1,18	0,9	0,10	0,07	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.5

T_t/T_k	T_{ff}/T_k	T/T_{ad}	ξ	$\lg T/T_k$	$\lg T/T_{ad}$	Δ
6445,6		10^4	0,01	3,81	4,00	0,8
102,14		10^2	0,1	2,01	2,00	0,08
29,30		25,00	0,2	1,47	1,40	0,07
$\Delta=10^{-2}$	6,74	11,11	0,3	0,83	1,04	0,05
	4,36	6,25	0,4	0,64	0,80	0,04
	3,09	4,00	0,5	0,49	0,60	0,03
	2,33	2,78	0,6	0,37	0,44	0,03
	1,82	2,04	0,7	0,26	0,31	0,03
	1,46	1,56	0,8	0,17	0,19	0,03
	1,20	1,23	0,9	0,08	0,09	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.6

V_s^T/V_{sk}	V_s^{ff}/V_{sk}	V_s/V_{sad}	ξ	$\lg V_s/V_{sk}$	$\lg V_s/V_{sad}$	Δ
80,03		10^2	0,01	1,90	2,00	0,8
10,07		10	0,1	1,00	1,00	0,08
5,40		5,00	0,2	0,73	0,70	0,07
	2,60	3,33	0,3	0,41	0,52	0,05
	2,09	2,50	0,4	0,32	0,40	0,04
	1,76	2,00	0,5	0,24	0,30	0,03
$\Delta=10^{-2}$	1,53	1,67	0,6	0,18	0,22	0,03
	1,35	1,43	0,7	0,13	0,15	0,03
	1,21	1,25	0,8	0,08	0,10	0,03
	1,10	1,11	0,9	0,04	0,04	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.7

$W_{r\varphi}^T/W_{r\varphi k}$	$W_{r\varphi}^{ff}/W_{r\varphi k}$	$W_{r\varphi}/W_{r\varphi}^{ad}$	ξ	$\lg W_{r\varphi}/W_{r\varphi k}$	$\lg W_{r\varphi}/W_{r\varphi}^{ad}$	Δ
$3{,}3.10^6$		$1{,}4999.10^6$	0,01	6,52	6,18	0,8
3301,0		1498,3	0,1	3,52	3,18	0,08
411,51		186,84	0,2	2,61	2,27	0,07
	52,13	54,90	0,3	1,72	1,74	0,05
	21,62	22,78	0,4	1,33	1,38	0,04
	10,80	11,74	0,5	1,03	1,07	0,03
$\Delta=10^{-2}$	6,04	6,30	0,6	0,78	0,80	0,03
	3,63	3,74	0,7	0,56	0,57	0,03
	2,30	2,31	0,8	0,36	0,36	0,03
	1,50	1,46	0,9	0,18	0,16	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.8

P_T/P_k	P_{ff}/P_k	P/P_{ad}	ξ	lgP/P_k	lgP/P_{ad}	Δ
$4{,}09.10^{10}$		$1{,}4999.10^{10}$	0,01	10,61	10,18	0,8
$3{,}25.10^{5}$		$1{,}4993.10^{5}$	0,1	5,51	5,18	0,08
9453,30		4671,00	0,2	3,98	3,67	0,07
	743,48	6,	0,3	2,87	2,78	0,05
	161,85	142,39	0,4	2,21	2,15	0,04
	49,10	46,96	0,5	1,69	1,67	0,03
$\Delta=10^{-2}$	18,31	17,50	0,6	1,26	1,24	0,03
	7,84	7,63	0,7	0,89	0,88	0,03
	3,90	5,61	0,8	0,59	0,56	0,03
	1,88.	1,80	0,9	0,27	0,25	0,03
	1,00	1,00	1,0	0,00	0,00	0,03

Table No.9

τ_T/τ_k	τ^{ff}/τ_k	τ/τ_{ad}	ξ	$lg\tau/\tau_k$	$lg\tau/\tau_{ad}$	Δ
517,15		150,00	0,01	2,71	2,18	0,8
32,73		14,19	0,1	1,51	1,18	0,08
14,18		7,47	0,2	1,15	0,87	0,07
	1,07	4,94	0,3	0,03	0,69	0,05
	1,07	3,64	0,4	0,03	0,56	0,04
	1,06	2,93	0,5	0,03	0,47	0,03
$\Delta=10^{-2}$	1,05	2,27	0,6	0,03	0,36	0,03
	1,04	1,83	0,7	0,02	0,26	0,03
	1,03	1,48	0,8	0,02	0,17	0,03
	1,02	1,18	0,9	0,01	0,07	0,03
	1,00	1,00	1,0	0,01	0,00	0,03

Table No.10

$(\Sigma_k/\Sigma_0)_\tau \cdot 10^{-2}$	$(\Sigma_k/\Sigma_0)_{ff} \cdot 10^{-4}$	Σ_{ad}/Σ_a	t_ϕ[d]
1,12	9,30	0,86	20
0,61	3,62	0,80	30
0,40	1,85	0,74	40
0,28	1,10	0,69	50
0,22	0,72	0,64	60
0,17	0,50	0,59	70
0,14	0,37	0,55	80
0,12	0,28	0,51	90
0,10	0,22	0,47	100